Final Exam for MAT 2377 3X (Spring 2011)
Probability and Statistics for Engineers.

Multiple Choice Questions

1. Solve
$$n \geq \left[\frac{z_{.025}\sigma}{E}\right]^2 = \left[\frac{(1.96)(1.6)}{0.5}\right]^2 = 39.3.$$
We need $n = 40$ observations.

2. Let X be the number of fluorescent lights that have a useful life of at least 500 hours among $n = 20$. X has a binomial distribution with $n = 20$ and $p = 0.9$. We want
$$P(X \leq 18) = 0.6083.$$

[1] 3. Since $(\overline{X} - 10)/(S/\sqrt{15}$ has a t distribution with $\nu = 14$ degrees of freedom, then $c = -t_{.05,14} = -1.761$.

4. A 95% confidence interval for the true proportion of helmets of this type that would show damage is
$$\hat{p} \pm z_{.025}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \frac{18}{50} \pm 1.96\sqrt{\frac{(18/50)(1-18/50)}{50}} = [0.227; 0.493]$$

5. Let X be the weight (in ounces) of a box. We want
$$P(X < 12) = \Phi\left(\frac{12-12.2}{\sqrt{0.0036}}\right) = \Phi(-3.33) = 0.0004.$$

6. See Question 7 of assignment 1.
7. Let A be the event that the part has a coarse edge condition and B that the depth of bore is above target. We want
$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{15/200}{25/200} = \frac{15}{25}$$
8. $P(A \cap B) = 20/200 = 0.1$, but $P(A)P(B) = (45/200)(110/200) \neq 0.12375$. Since $P(A \cap B) \neq P(A)P(B)$, then A and B are not independent.
9. Ignore the question.
10. We want to test $H_0 : \mu = 3,500$ against $H_1 : \mu \neq 3,500$, under the conditions of a normal population with σ known (i.e. $\sigma = 57$). The observed value of the test statistic is
$$z_0 = \frac{\overline{x} - 3,500}{\sigma/\sqrt{n}} = \frac{3,465 - 3,500}{57/\sqrt{25}} = -3.07.$$
The p-value is
$$p = 2\,P(Z > |z_0|) = 2\,(1 - \Phi(-3.07)) = 2(1 - .9989) = 0.0022.$$
Since $p < 0.05$, we can reject H_0. At a level of significance of 5%, we can conclude that the mean $\mu \neq 3,500$.
11. A point estimate for the mean is $\hat{\mu} = \overline{x} = 323.24$ and the estimated standard error of the estimate is $s/\sqrt{n} = 9.991482/\sqrt{15} = 2.579790$.

[**1**] 12. **Conditions :** normal population with σ unknown.

A 95% confidence interval for μ is
$$\overline{x} \pm t_{0.025,14}\frac{s}{\sqrt{n}} = 323.24 \pm 2.145\,\frac{9.991482}{\sqrt{15}} = [317.71, 328.77]$$
13. Let E be the event that the voltage regulator performs according to specifications. We want
$$\begin{aligned} P(A|E) &= \frac{P(A \cap E)}{P(E)} \\ &= \frac{P(E|A)P(A)}{P(E|A)P(A) + P(E|B)P(B) + P(E|C)P(C)} \\ &= \frac{(.95)(.3)}{(.95)(.3) + (.8)(.6) + (.65)(.1)} \\ &= 0.3958 \end{aligned}$$

14. Ignore this question.